Looking at $H(\text{div})$ conforming FEM, it turns out that the after defining the relevant vector basis functions on the reference element $\hat{\Omega}$, that the transition to a mesh is not trivial. The key is that we must preserve the normal components of the basis functions when moving from $\hat{\Omega} \to \Omega$, where $\Omega := F(\Omega)$ with $F(\hat{x}) = \mathbf{A}\hat{x} + b$ for now for $\mathbf{A}$ a matrix, b a vector, and $\hat{x} \in \hat{\Omega}$. For the sake of simplicity, I will omit the vector marks.

The correct mapping needed is called the Piola transform, and is defined to be

$$u(x) := \frac{1}{|\mathbf{A}|}\mathbf{A}\hat{u} \cdot F^{-1}(x)$$

where $\hat{u} : \hat{\Omega} \to \mathbb{R}^n$.

Let us check that the finite element unisolvence conditions can be satisfied on the edges:

$$\int_E u \cdot nv \, dx = \int_{\hat{E}} \hat{u} \cdot \hat{n}\hat{v} \, d\hat{x}$$

where $\hat{E}, E = F(\hat{E})$ edges and $v, \hat{v}$ sufficiently smooth functions (e.g. $H^{1/2}$, but we will not worry about this) on said edges. One has, using the change of variables $x = F(\hat{x}) \implies dx = |DF| \, d\hat{x}$ with $|DF| = |\mathbf{A}|$,

$$\begin{aligned}
\int_E u \cdot nv \, dx &= \int_E \frac{1}{|\mathbf{A}|}\mathbf{A}\hat{u} \cdot F^{-1}(x) \cdot (\mathbf{A}^{-1}\hat{n})v(x) \, dx \\
&= \int_{\hat{E}} \frac{1}{|\mathbf{A}|}\mathbf{A}\hat{u}(\hat{x}) \cdot (\mathbf{A}^{-1}\hat{n})v(F^{-1}(\hat{x})) \, |\mathbf{A}| \, d\hat{x} \\
&= \int_{\hat{E}} \hat{u}(\hat{x}) \cdot \hat{n}\hat{v}(\hat{x}) \, d\hat{x}.
\end{aligned}$$

Another condition which is of importance

$$\int_\Omega \text{div} uv \, dx = \int_{\hat{\Omega}} \hat{\text{div}} \hat{u}\hat{v} \, d\hat{x}$$

which can be seen with

$$\begin{aligned}
\int_\Omega \text{div} uv \, dx &= \int_\Omega \text{tr}\left(\nabla_x \left(\frac{1}{|\mathbf{A}|}\mathbf{A}\hat{u} \cdot F^{-1}(x)\right)\right) v(x) \, dx \\
&= \int_\Omega \frac{1}{|\mathbf{A}|} \text{tr}\left(\mathbf{A}\nabla_x \hat{u} \cdot F^{-1}(x)\mathbf{A}^{-1}\right) v(x) \, dx \\
&= \int_\Omega \frac{1}{|\mathbf{A}|} \text{tr}\left(\nabla_x \hat{u} \cdot F^{-1}(x)\right) v(x) \, dx \\
&= \int_\Omega \frac{1}{|\mathbf{A}|} \text{tr}\left(\nabla_{\hat{x}} \hat{u}(\hat{x})\right) v(F^{-1}(x)) \, |\mathbf{A}| \, d\hat{x} = \int_{\hat{\Omega}} \hat{\text{div}} \hat{u}\hat{v} \, d\hat{x}.
\end{aligned}$$

These are nice properties, and are not hard to verify for the linear transform, but as of writing, the transition to nonlinear diffeomorphisms are... not trivial for me to show. In particular, since the Jacobian and determinants now vary, the Piola transform is

$$u(x) := \frac{1}{J} DF(F^{-1}x)\hat{u}(F^{-1}(x))$$

where J is the determinant of F evaluated at $F^{-1}x$. Taking derivatives of this is... tough to say the least, but it seems the correct way is to use a FEEC approach... to be continued in a week or so.